Centre Number						Candidate Number				
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Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 – 5	
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12 – 13	
14	
TOTAL	

AQA

Level 2 Certificate in Further Mathematics
June 2012

Further Mathematics 8360/1
Level 2
Paper 1 Non-Calculator

Tuesday 29 May 2012 1.30 pm to 3.00 pm

For this paper you must have:
- mathematical instruments.

You may **not** use a calculator.

Time allowed
- 1 hour 30 minutes

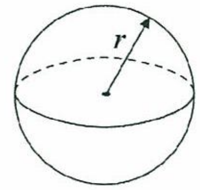
Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 70.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer booklet.

Formulae Sheet

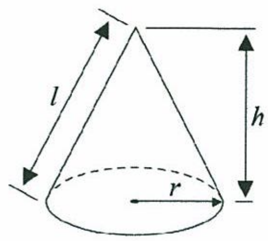
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$

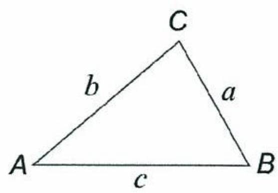


Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$



In any triangle *ABC*



Area of triangle $= \frac{1}{2}ab\sin C$

Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Trigonometric Identities

$\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$ $\qquad \sin^2\theta + \cos^2\theta \equiv 1$

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Answer **all** questions in the spaces provided.

1 $f(x) = 2x^2 + 7$ for all values of x.

1 (a) What is the value of $f(-1)$?

Answer $2(-1)^2 + 7 = 2(1) + 7 = 9$ *(1 mark)*

1 (b) What is the range of $f(x)$?

Answer $f(x) \geqslant 7$ *(1 mark)*

$x^2 \geqslant 0$ for all x, so $2x^2 \geqslant 0$ and $2x^2 + 7 \geqslant 7$

2 $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$

Work out the matrix **AB**.

$$AB = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 5 \\ 4 \end{pmatrix} = \begin{pmatrix} 2(5)+0(4) \\ 1(5)+3(4) \end{pmatrix} = \begin{pmatrix} 10 \\ 17 \end{pmatrix}$$

AB = $\begin{pmatrix} 10 \\ 17 \end{pmatrix}$ *(2 marks)*

4

3 Work out the greatest integer value of x that satisfies the inequality $3x + 10 < 1$

$x < \frac{1-10}{3} \Rightarrow x < -3$

$\therefore$ Greatest integer value for x $= -4$

Answer $x = -4$. *(2 marks)*

4 (a) Factorise fully $2x^2 - 2x - 40$

$(2x + 8)(x - 5) \equiv 2(x+4)(x-5)$

Answer $2(x+4)(x-5)$ *(3 marks)*

4 (b) Factorise fully $(x + y)^2 + (x + y)(2x + 5y)$

$(x+y)[(x+y) + (2x+5y)]$

$= (x+y)(3x + 6y)$

$= 3(x+y)(x+2y)$

Answer $3(x+y)(x+2y)$ *(3 marks)*

Do not write outside the box

5 Simplify $(2cd^4)^3$

$2^3c^3(d^4)^3 = 8c^3d^{(4\times3)} = 8c^3d^{12}$

Answer $8c^3d^{12}$ (2 marks)

6 Solve the simultaneous equations

$$2y = 3x + 4$$

$$2x = -3y - 7$$

Do **not** use trial and improvement.

$2y - 3x = 4 \quad \dots \; ①$

$2x + 3y = -7 \quad \dots \; ②$

① × 2: $4y - 6x = 8 \quad \dots \; ③$

② × 3: $6x + 9y = -21 \quad \dots \; ④$

③ + ④: $13y = -13 \Rightarrow y = -1$

In ①, $x = \dfrac{2y-4}{3} = \dfrac{2(-1)-4}{3} = \dfrac{-6}{3} = -2$

$\therefore (x, y) = (-2, -1)$

Answer $(x, y) = (-2, -1)$ (4 marks)

14

Turn over ▶

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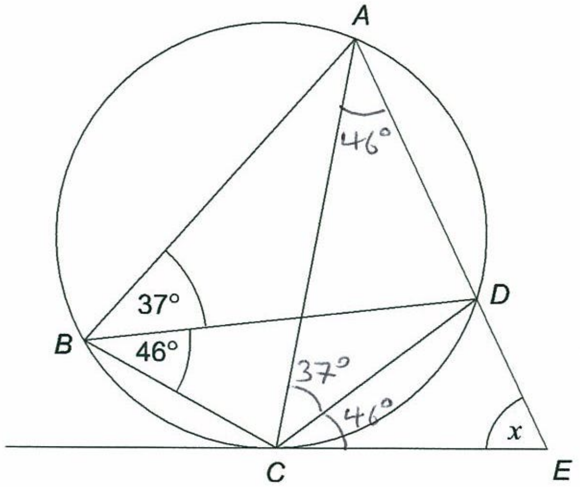
7 The diagram shows a cyclic quadrilateral *ABCD*.

ADE is a straight line.
CE is a tangent to the circle.



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Work out the size of angle x.

CAD = 46° and ACD = 37° since angles in the same segment are equal.
DCE = 46° since the angle in the opposite segment must be equal (i.e. CBD = DCE).
$x = 180 - 2(46) - 37 = 51°$ — Angles of a triangle add to 180°

$x =$ 51° degrees *(3 marks)*

Do not write outside the box

8 A curve has equation $y = x^3 + 5x^2 + 1$

8 (a) When $x = -1$, show that the value of $\frac{dy}{dx}$ is -7.

$\frac{dy}{dx}$ or $f'(x) = 3x^2 + 10x$

and $f'(-1) = 3(-1)^2 + 10(-1)$

$= 3(1) - 10 = -7$.

(2 marks)

8 (b) Work out the equation of the tangent to the curve $y = x^3 + 5x^2 + 1$ at the point where $x = -1$

$f(-1) = (-1)^3 + 5(-1)^2 + 1 = -1 + 5 + 1 = 5$

So tangent in question passes through $(-1, 5)$ with gradient given by $f'(-1) = -7$.

Equation of tangent is given by $y = -7x + c$ passing through $(-1, 5)$. $5 = -7(-1) + c \Rightarrow c = 5 - 7 = -2$

Answer $y = -7x - 2$ *(4 marks)*

Turn over for the next question

9

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9 Write this ratio in its simplest form

$$\sqrt{12} : \sqrt{48} : \sqrt{300}$$

$\sqrt{12} : \sqrt{4 \times 12} : \sqrt{25 \times 12}$

$= \sqrt{12} : 2\sqrt{12} : 5\sqrt{12}$

$= 1 : 2 : 5$

Answer 1 : 2 : 5 *(3 marks)*

10 The n^{th} term of the linear sequence 2 7 12 17 ... is $5n - 3$

A new sequence is formed by squaring each term of the linear sequence and adding 1 .

Prove algebraically that **all** the terms in the new sequence are multiples of 5 .

Multiples of 5 can be expressed as $5n$ where n is an integer.

n^{th} term of new sequence is given by $(5n-3)^2 + 1$

$= (5n-3)(5n-3) + 1 = 25n^2 - 30n + 9 + 1$

$= 25n^2 - 30n + 10 = 5(5n^2 - 6n + 2)$

$\therefore$ Since n must be an integer, so too must $5n^2 - 6n + 2$ & so $5(5n^2 - 6n + 2)$ will be a multiple of 5.

(4 marks)

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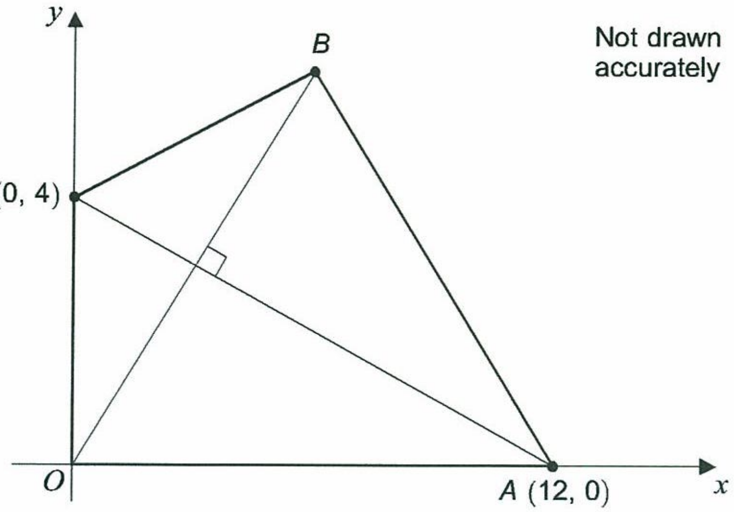
11 *OABC* is a kite.



11 (a) Work out the equation of *AC*.

$y = mx + c$ where $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-4}{12-0} = -\frac{1}{3}$

and $c = 4$ (i.e. the y-intercept)

$\therefore y = -\frac{1}{3}x + 4$ or $3y + x = 12$

Answer $y = -\frac{1}{3}x + 4$ *(2 marks)*

11 (b) Work out the coordinates of *B*.

As OABC is a kite, AC is the perpendicular bisector of OB whose equation must therefore be given by $y = 3x$. If (a, b) is the intersection point for lines AC and OB, then a must be the solution to x in $3x = -\frac{1}{3}x + 4$, i.e. $\frac{10}{3}x = 4 \Rightarrow x = \frac{4}{10/3} = 4 \times \frac{3}{10} = \frac{12}{10} = \frac{6}{5}$ or 1.2 and $y = 3\left(\frac{6}{5}\right) = \frac{18}{5}$ or 3.6. $\therefore (a, b) = (1.2, 3.6)$

Finally, since (a, b) is the midpoint of OB, coordinates of B are given by $(2a, 2b) = (2.4, 7.2)$

Answer (2.4 , 7.2) *(6 marks)*

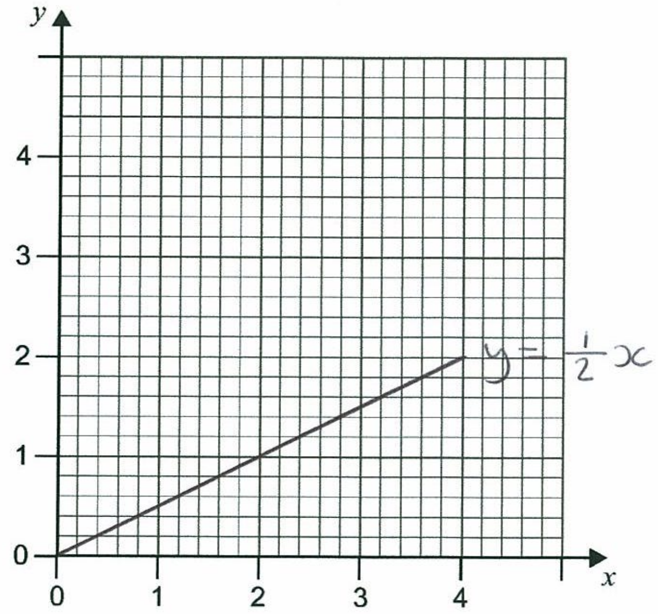
15

Turn over ▶

12 (a) A graph passes through (0, 0).

The rate of change of y with respect to x is always $\frac{1}{2}$.

Draw the graph of y for values of x from 0 to 4.

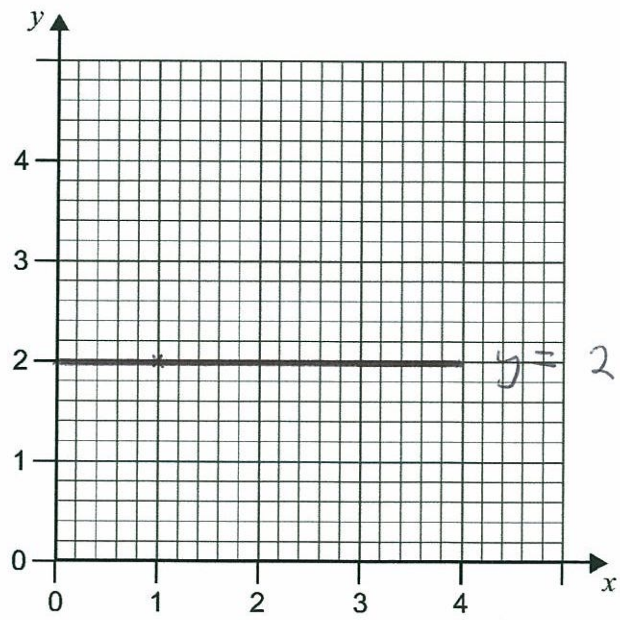


(1 mark)

12 (b) A graph passes through (1, 2).

The rate of change of y with respect to x is always 0.

Draw the graph of y for values of x from 0 to 4.



(1 mark)

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12 (c) $y = 2x^3 + ax$, where a is a constant.

The value of $\frac{dy}{dx}$ when $x = 2$ is twice the value of $\frac{dy}{dx}$ when $x = -1$

Work out the value of a.

$\frac{dy}{dx}$ or $f'(x) = 6x^2 + a$

$f'(2) = 2f'(-1)$

$\Rightarrow 6(2)^2 + a = 2[6(-1)^2 + a]$

$\Rightarrow 24 + a = 2(6 + a)$

$\Rightarrow 24 + a = 12 + 2a$

$\Rightarrow a = 12$

$a =$ 12 *(5 marks)*

Turn over for the next question

7

Do not write outside the box

13 Simplify $\dfrac{x^2+4x-12}{x^2-25} \div \dfrac{x+6}{x^2-5x}$

$$\frac{(x-2)\cancel{(x+6)}}{(x+5)\cancel{(x-5)}} \times \frac{x\cancel{(x-5)}}{\cancel{x+6}}$$

Difference of two squares

$$= \frac{x(x-2)}{x+5}$$

Answer $\dfrac{x(x-2)}{x+5}$ *(5 marks)*

14 $x^{\frac{3}{2}} = 8$ where $x > 0$ and $y^{-2} = \dfrac{25}{4}$ where $y > 0$

Work out the value of $\dfrac{x}{y}$.

$$x^{\frac{3}{2}} = 8 \Rightarrow (\sqrt{x})^3 = 8 \Rightarrow x = \overbrace{(\sqrt[3]{8})}^{\text{or } 8^{\frac{2}{3}}}{}^2 = 4$$

$$y^{-2} = \frac{25}{4} \Rightarrow \frac{1}{y^2} = \frac{25}{4} \Rightarrow y^2 = \frac{1}{\left(\frac{25}{4}\right)} = \frac{4}{25}$$

$$\Rightarrow y = \sqrt{\frac{4}{25}} = \frac{2}{5}$$

$$\therefore \frac{x}{y} = \frac{4}{2/5} = 4 \times \frac{5}{2} = \frac{20}{2} = 10$$

$\dfrac{x}{y} =$ 10 *(5 marks)*

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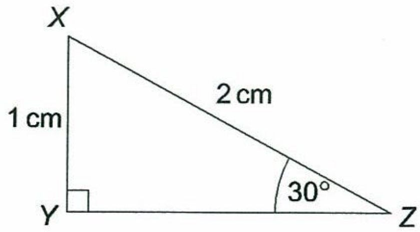
15 (a) *XYZ* is a right-angled triangle.



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Use triangle *XYZ* to show that $\sin 60° = \frac{\sqrt{3}}{2}$

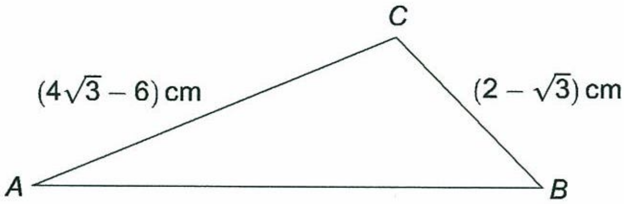
$\sin\theta = H : O$ or $\frac{O}{H}$. $Y\hat{X}Z = 180 - 90 - 30 = 60°$

$\sin 60° = \frac{O}{H} = \frac{YZ}{2}$ where $YZ = \sqrt{2^2 - 1^2} = \sqrt{3}$

$\therefore \sin 60° = \frac{\sqrt{3}}{2}$

(2 marks)

15 (b) Triangle *ABC* has an obtuse angle at *C*.



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Given that $\sin A = \frac{1}{4}$, use triangle *ABC* to show that angle $B = 60°$

From part (a) $\sin^{-1}\frac{\sqrt{3}}{2} = 60°$

From the sine rule, $\frac{\sin A}{2-\sqrt{3}} = \frac{\sin B}{4\sqrt{3} - 6}$

$\Rightarrow \frac{1}{4(2-\sqrt{3})} = \frac{\sin B}{4\sqrt{3} - 6}$

$\Rightarrow \sin B = \frac{4\sqrt{3} - 6}{4(2-\sqrt{3})} = \frac{2\sqrt{3} - 3}{2(2-\sqrt{3})} = \frac{(2\sqrt{3}-3)(2+\sqrt{3})}{2(2-\sqrt{3})(2+\sqrt{3})}$

$= \frac{4\sqrt{3} + \cancel{6} - \cancel{6} - 3\sqrt{3}}{2(4-3)} = \frac{\sqrt{3}}{2}$

$\therefore$ Since $\sin B = \frac{\sqrt{3}}{2}$, $B = \sin^{-1}\frac{\sqrt{3}}{2} = 60°$

(6 marks)

18

Turn over ▶

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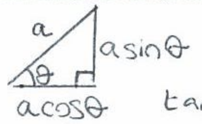
16 Prove that $\tan\theta + \dfrac{1}{\tan\theta} \equiv \dfrac{1}{\sin\theta\cos\theta}$ *



$\tan\theta = \dfrac{O}{A} = \dfrac{\cancel{a}\sin\theta}{\cancel{a}\cos\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ *

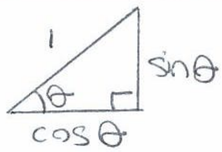
$\Rightarrow \tan\theta + \dfrac{1}{\tan\theta} \equiv \dfrac{\sin\theta}{\cos\theta} + \dfrac{1}{\left(\dfrac{\sin\theta}{\cos\theta}\right)}$

$\equiv \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} \equiv \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$\equiv \dfrac{1}{\sin\theta\cos\theta}$

(3 marks)

N.B: $\sin^2\theta + \cos^2\theta \equiv 1$



END OF QUESTIONS

From Pythagoras' theorem, $1^2 = \cos^2\theta + \sin^2\theta$

i.e. $\sin^2\theta + \cos^2\theta \equiv 1$

3

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